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Magneto-thermo-elastokinetics of geometrically nonlinear laminated composite plates. Part 1: foundation of the theory

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Abstract

A fully coupled magneto-thermo-elastokinetic model of laminated composite, finitely electroconductive plates incorporating geometrical nonlinearities and subjected to a combination of magnetic and thermal fields, as well as carrying an electrical current is developed. In this context, the first-order transversely shearable plate theory in conjunction with von-Kármán geometrically nonlinear strain concept is adopted. Related to the distribution of electric and magnetic field disturbances within the plate, the assumptions proposed by Ambartsumyan and his collaborators are adopted. Based on the electromagnetic equations (i.e. the ones by Faraday, Ampère, Ohm, Maxwell and Lorentz), the modified Fourier's law of heat conduction and on the elastokinetic field equations, the 3-D coupled problem is reduced to an equivalent 2-D one. The theory developed herein provides a foundation for the investigation, both analytical and numerical, of the interacting effects among the magnetic, thermal and elastic fields in multi-layered thin plates made of anisotropic materials.

1. Introduction

With the continuous advances in composite material processing and manufacturing, anisotropic composite thin-walled structures in the form of beams, plates and shells are likely

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Nomenclature

$\mathbf{B}$, $\mathbf{B}_0$, $\mathbf{b}$	magnetic induction vector, the primary and disturbed counterparts, respectively
C_t, α_{ij}	thermal coefficients
$\mathbf{E}$, $\mathbf{e}$	electric field vector and its disturbance counterpart, respectively
$2h$	plate thickness
$\mathbf{H}$, $\mathbf{H}_0$, $\mathbf{h}$	magnetic field vector, its primary and disturbance counterparts, respectively
I_0, I_2	inertia coefficients, defined in Eq. (20)
$\mathbf{J}$, $\mathbf{J}_0$, $\mathbf{j}$	electrical current density vector, its primary and disturbance counterparts, respectively
k_{ij}	thermal conductivity, see Eq. (1f)
$2\ell_1$, $2\ell_2$	plate length and width, respectively
N_L	number of the constituent layers of the plate
$\mathbf{n}$	unit vector of the external normal of the deformed configuration
S_{ij}	Second Piola–Kirchhoff stress tensor
$\mathbb{T}_{ij}$, $(\mathbb{T})_e$	magnetic Maxwell's stress tensors within the plate and in the vacuum, respectively
V_i, v_i	3-D and 2-D displacement components
x_i	Cartesian orthogonal coordinates
z_k, z_{k+1}	x_3 coordinate of the lower and upper surfaces of the kth layer, respectively
α_s^+, α_s^-	heat transfer coefficients at the plate upper and lower surfaces $x_3 = \pm h$, respectively
β_1, β_2	rotations of the transverse normal of the plate mid-plane about x_2- and negative x_1-axis, respectively, see Fig. 1
Θ	temperature beyond the reference temperature T_0
Θ_0, Θ_1	thermal field variables defined in Eqs. (54a,b)
$\Theta_s^\pm$	the surface temperature at $x_3 = \pm h$, respectively
δ_{ij}	the Kronecker delta
μ_0	magnetic permeability in vacuum, $4\pi \times 10^{-7}\,\mathrm{H/m}$
ρ_0	mass density (per volume) of the plate
ϑ	ply orientation angle, see Fig. 1
φ, ψ	e_1 and e_2 components of the disturbed electric field, respectively
χ	the thickness component of the disturbed magnetic field $\mathbf{h}$, see Eq. (28c)
$\Phi_1^{(e)}$	the disturbed magnetic potential outside the plate
$\mathscr{M}_0[g_{\alpha\beta}]$	$\sum_{i=1}^{N_L} g_{\alpha\beta}^{(i)}/N_L$
$\mathscr{M}_1[g_{\alpha\beta}]$	$\sum_{i=1}^{N_L} g_{\alpha\beta}^{(i)}(z_{i+1}+z_i)/(2N_L)$
$\mathscr{M}_2[g_{\alpha\beta}]$	$\sum_{i=1}^{N_L} g_{\alpha\beta}^{(i)}(z_{i+1}^2+z_{i+1}z_i+z_i^2)/(3N_L)$
$\mathscr{M}_4[g_{\alpha\beta}]$	$\sum_{i=1}^{N_L} g_{\alpha\beta}^{(i)}(z_{i+1}^4+z_{i+1}^3 z_i+z_{i+1}^2 z_i^2+z_{i+1}z_i^3+z_i^4)/(5N_L)$
$\nabla(\cdot)$	3-D gradient operator
$\nabla^2(\cdot)$	3-D Laplace operator

to play a crucial role in the construction, to name only a few, of aerospace vehicles, spacecraft and nuclear reactors. One recently proposed concept of multi-functional materials/structures featuring interactive elastic, thermal, magnetic and electric fields, etc. is likely to bring a new dimension to the application of composite materials/structures. On one hand, such a concept emphasizes highly synergistic design paradigm, so that many redundant components can be eliminated; on the other hand, electro- or magneto-active constituent materials bring configurable or extended functionalities to the structure. In fact, the underlying idea of the concept of multi-functional materials/structures is to exploit multiple properties of materials or structures in such a way that besides its major designated functionality, the same structural component should accomplish at least one more task. As an example, besides their primary load carrying capability, the electro/magneto-active structural components can be used in health monitoring of the host structure, or feedback control, etc.

As a consequence, in order to be able to efficiently design and exploit the multi-functional structures, a better understanding of their behavior when subjected to fully interactive actions

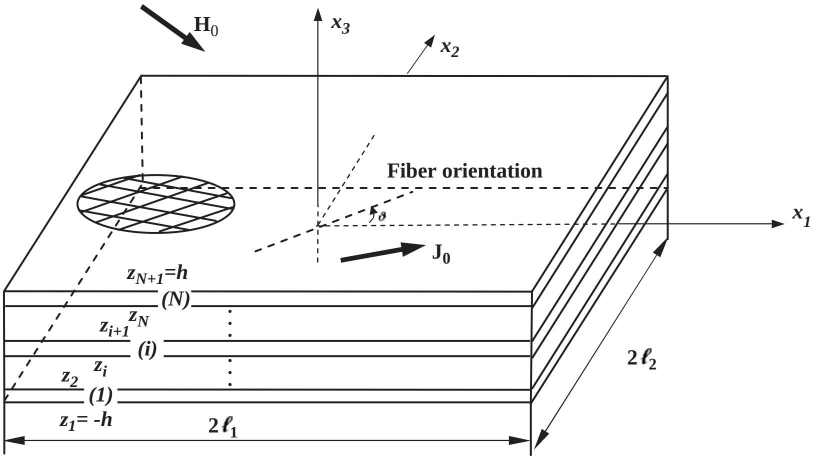

Fig. 1. Geometry of a laminated composite plate immersed in a magnetic field and carrying electric current.

among mechanical, thermal, electrical and magnetic fields becomes imperative. During the last few decades, much research work has been conducted on thermo-elastic, electro-elastic, magneto-elastic and 3-D magneto-thermo-elastic interactions (see e.g., Refs. [1–9]). However, little work has been devoted to thin-walled structures made of anisotropic materials, exposed to magneto-thermo-elastic fields.

In this article, a geometrically nonlinear magneto-thermo-elastokinetic theory of finitely electroconductive, multi-layered composite plates based on the first-order transversely shearable plate model will be developed. New issues raised by the structural lamination, such as the determination of the induced magnetic field at the layer interfaces are addressed, and the induced magnetic field outside the plate is determined. The results supplied herein are expected to provide a foundation for the investigation of the interactive effects among the magnetic, thermal and elastic fields in thin-walled structures and of the possibility to apply the magneto-thermo-elastic tailoring.

The organization of this article is as follows: at first, the geometrical description of the problem is given in Section 2. Then as a starting point, the 3-D Maxwell equations, the heat conduction laws and the balance of the momentum equations, as well as the associated boundary/jump conditions are given in Section 3. Based on these 3-D equations, the corresponding 2-D equations are derived in three separate parts, namely, elastic, electrodynamic and thermal ones. In the elastic part (Section 4.1), based on a first-order transversely shearable plate theory [10,11], the 2-D equations of motion in the presence of generalized forces of electrodynamic and thermal origins are derived. In the electrodynamic part (Section 4.2), the assumptions due to Ambartsumyan et al. [1,2] are considered, then the equations of motion governing the induced electromagnetic fields inside/outside the plate are derived in Sections 4.2.2 and 4.2.1, respectively. To facilitate the sequential reference of equations, the induced magnetic field outside the plate is tackled before the problem of the induced magnetic field inside the plate. The 2-D thermal equations are derived in Section 4.3, and the 2-D electrodynamic loads of Lorentz origin are summarized in Section 4.4. The special case studies and the numerical analysis will be conducted in Part 2.

2. Geometrical description of the problem

Consider an elastic plate consisting of N_L electrically and thermally conductive, orthotropic layers with symmetric lay-up configuration (for its geometric characteristics, see Fig. 1), immersed in an applied static magnetic field $\mathbf{H}_0^{ex}$, and carrying an electric current $\mathbf{J}_0$ having a planar distribution $(J_{01}, J_{02}, 0)$. It is assumed that each constituent layer is homogeneous, of equal and uniform thickness. It is further assumed that the principal axes of electric conductivity, heat conduction and thermal expansion coincide with the directions of principal elastic orthotropy of each layer.

The points of the non-deformed plate configuration are referred to the 3-D Cartesian coordinate system x_i $(i = 1, 2, 3)$, where (x_1, x_2) are the in-plane coordinates associated with the points of the non-deformed mid-plane Ω_0 of the plate, while x_3 is the thickness coordinate.

3. Field equations

In the modeling of the plate, the geometric nonlinearities are incorporated. While the elastic, thermal and the electromagnetic fields themselves can be determined individually from the elastic, thermal and the Maxwell's equations, the interaction of these three fields can generate new phenomena. As it will become evident in the sequel, the electromagnetic field influences the thermo-elastic field by entering the elastic stress equations of motion as the magnetic body forces (referred to as the Lorentz's pondermotive force) and the Maxwell's stress jumps on the plate surfaces, whereas the elastic and thermal fields in their turn influence the electromagnetic field by modifying Ohm's law.

In order to be reasonably self-contained, in what follows, we will summarize the electromagnetic equations, as well as the equations of motion of a 3-D elastic medium. Expressed in MKS (meter–kilogram–second) system of units and in the absence of electric free charges, the relevant equations are [3,12,13]:

$$\text{Faraday's Law: } \operatorname{curl}\mathbf{E} = -\dot{\mathbf{B}}, \quad \text{Ampère's Law: } \operatorname{curl}\mathbf{H} = \mathbf{J}, \tag{1a,b}$$

$$\text{Gauss' Law: } \operatorname{div}\mathbf{D} = 0, \quad \text{Conservation of flux: } \operatorname{div}\mathbf{B} = 0. \tag{1c,d}$$

Equations of motion of geometrically nonlinear 3-D elastic bodies are in Lagrangian description:

$$\left[S_{jr}(\delta_{ir} + V_{i,r})\right]_{,j} + f_i = \rho_0 \ddot{V}_i. \tag{1e}$$

The modified Fourier's law of heat conductions:

$$k_{ij}\Theta_{,ij} - C_t\dot{\Theta} + W^t = 0. \tag{1f}$$

In these equations, as well as in the remaining ones of this article (Part 1), unless otherwise specified, the Einstein summation convention over a repeated index is implied, and the Latin indices range from 1 to 3, while Greek indices range from 1 to 2. In addition, partial differentiation is denoted by a comma, $(\cdot)_{,i} \equiv \partial(\cdot)/\partial x_i$, and the time derivative(s) by overdot(s): $\dot{(\cdot)} \equiv \partial(\cdot)/\partial t$, $\ddot{(\cdot)} \equiv \partial^2(\cdot)/\partial t^2$.

In Eq. (1e), **E** and **H** are the electric and magnetic field vectors, respectively; **J** is the current–density vector, **B** is the magnetic induction vector and ρ_0 is the mass per unit volume of the elastic solid in the underformed state; δ_{ij} is the Kronecker delta, $S_{ij}(\equiv S_{ji})$ are the components of the second Piola–Kirchhoff stress tensor, $\mathbf{V}[V_1, V_2, V_3]$ is the displacement vector of the points of the 3-D elastic medium. In connection with Eq. (1f), $W^t = J_i(\mathbb{G}^{-1})_{ij}J_j$ denotes the intensity of heat supply due to the Joule's heat effect, and $\mathbb{G}$ is the electric conductivity matrix (second-order tensor). In an off-axis system, $\mathbb{G}$ is given by

$$\mathbb{G} \equiv \begin{bmatrix} g_{11} & g_{12} & 0 \\ g_{12} & g_{22} & 0 \\ 0 & 0 & g_{33} \end{bmatrix}. \tag{2}$$

Herein, $g_{11} = g_1\cos^2\vartheta + g_2\sin^2\vartheta$, $g_{22} = g_2\cos^2\vartheta + g_1\sin^2\vartheta$, $g_{12} = (g_1 - g_2)\sin\vartheta\cos\vartheta$, $g_{33} = g_3$ while g_i $(i = 1, 2, 3)$ are the components of electric conductivity along the principal axes direction; ϑ is the ply angle (see Fig. 1 for its definition); $\Theta = T - T_0$ is the increase of the temperature field beyond a stress-free reference temperature T_0; k_{ij} and C_t are the thermal coefficients, in which the second-order tensor k_{ij} follows the same rule of transformation as g_{ij} in Eq. (2). In Eq. (1e), f_i are the components of the Lorentz force vector **f** per unit volume, which can be expressed as

$$\mathbf{f} = \mathbf{J} \times \mathbf{B}. \tag{3}$$

For an anisotropic, linear and magnetizationless material, the constitutive equations are

$$\mathbf{B} = \mu_0\mathbf{H}, \quad \mathbf{J} = \mathbb{G}(\mathbf{E} + \dot{\mathbf{V}} \times \mathbf{B}), \quad S_{ij} = C_{ijkl}(\varepsilon_{kl} - \alpha_{kl}\Theta). \tag{4a–c}$$

In these equations, μ_0 is the magnetic permeability in vacuum, C_{ijkl} are the elastic coefficients corresponding to the reference temperature, α_{kl} are the thermal coefficients. In Ohm's law Eq. (4b) and the elastic constitutive relations (4c), the effect of the temperature on the electric conductivity and the elastic constants is disregarded.

The constitutive relations (4c) in the off-axis coordinate system (x, y, z) are expressed as

$$\begin{Bmatrix} S_{11} \\ S_{22} \\ S_{33} \\ S_{23} \\ S_{13} \\ S_{12} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & 0 & 0 & Q_{16} \\ Q_{12} & Q_{22} & Q_{23} & 0 & 0 & Q_{26} \\ Q_{13} & Q_{23} & Q_{33} & 0 & 0 & Q_{36} \\ 0 & 0 & 0 & Q_{44} & Q_{45} & 0 \\ 0 & 0 & 0 & Q_{45} & Q_{55} & 0 \\ Q_{16} & Q_{26} & Q_{36} & 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} - \alpha_{11}\Theta \\ \varepsilon_{22} - \alpha_{22}\Theta \\ \varepsilon_{33} - \alpha_{33}\Theta \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} - \alpha_{12}\Theta \end{Bmatrix}. \tag{5}$$

Herein, Q_{ij}, $(i,j) = \overline{1, 6}$ are the transformed counterparts of the ones in the on-axis system. Transformation of α_{11}, α_{22} follows the same rule as that of g_{ij}, while $\alpha_{12} = 2(\alpha_1 - \alpha_2)\cos\vartheta\sin\vartheta$.

The preceding equations are associated with the inner domain occupied by the plate. For the domain outside the plate (considered to be the vacuum), the equations governing the electromagnetic field are given by

$$\operatorname{curl}\mathbf{H}^{(e)} = \mathbf{0}, \quad \operatorname{div}\mathbf{H}^{(e)} = 0, \quad \operatorname{curl}\mathbf{E}^{(e)} = -\dot{\mathbf{B}}^{(e)}, \tag{6a–c}$$

where the superscript "e" identifies the quantities associated with the domain outside the plate.

Finally, towards establishing the governing equations of electroconductive plates, it is recalled that in a magnetic field, the electromagnetically induced forces act also on the external surfaces of the electroconductive body. These forces are related to the Maxwell's stress tensor in the form

$$\mathbb{T}_{ij} = B_i H_j - \tfrac{1}{2}\mu_0 |\mathbf{H}|^2 \delta_{ij}, \tag{7}$$

where H_i are the components of the magnetic field $\mathbf{H}$.

At the external surfaces of the plate that separate two media with different electromagnetic properties, the related fields experience discontinuities that are specified by a number of boundary conditions. Restricting ourselves to the conditions that will be required in the following developments, these conditions are

$$\mathbf{n} \times [\mathbf{E} - \mathbf{E}^{(e)}] = \mathbf{0}, \quad \mathbf{n} \cdot [\mathbf{B} - \mathbf{B}^{(e)}] = 0, \tag{8a, b}$$

where $\mathbf{n}$ is the unit vector of the external normal of the plate surfaces. These conditions stipulate that the tangential components of $\mathbf{E}$ and the normal components of $\mathbf{B}$ are continuous at the media interfaces. For a perfectly electroconductive material, the jump condition

$$\mathbf{n} \times [\mathbf{H} - \mathbf{H}^{(e)}] = \mathbf{J}_s \tag{9}$$

will be used to determine the surface electric current vector $\mathbf{J}_s$, while for finitely electroconductive materials, $\mathbf{J}_s = 0$.

In the same context, the mechanical boundary conditions that should be fulfilled on the boundary surfaces $x_3 = \pm h$ of the plate are expressed as [6,13–15]

$$n_i(S_{ij} + S_{jr}V_{i,r} + \mathbb{T}_{ij}) = F_j + n_i(\mathbb{T}_{ij})_e, \tag{10}$$

where n_i are the components of the unit vector $\mathbf{n}$, while F_j are the components of the surface load vector $\mathbf{F}$ of mechanical origin.

The thermal boundary conditions on the external plate surfaces $x_3 = \pm h$ can be represented in the form [9,16]:

$$n_i k_{ij} \Theta_{,j} + \alpha_s(\Theta - \Theta_s) = 0, \tag{11}$$

where α_s is the surface heat transfer coefficient, (see e.g., Ref. [17, p. 8]) and Θ_s is the temperature quantity at the surface of the plate.

In Eqs. (1a–f), there are nonlinear terms that considerably complicate the investigation of an already intricate problem. At this point, one should distinguish three types of nonlinearities involved in Eqs. (1a–f), namely, (1) the structural nonlinearities, such as those involved in Eq. (1e) and in the kinematical relations between V_i and ε_{ij}, (2) the nonlinearities of purely electromagnetic origin, such as those involved in Eq. (1f), and (3) the nonlinearities of the mixed nature, such as those involved in the Ohm's law (i.e., Eq. (1c)), the boundary/jump conditions (i.e., Eqs. (8a–c), (10) and (11)). In the following treatment, only the geometrical nonlinearities will be retained. The nonlinear terms of electromagnetic and mixed origins, which appear in the previously indicated equations, will be linearized via the small disturbance concept.

4. 2-D magneto-thermo-elastic governing equations

The extended Hamilton's principle [18] is used to construct the first-order transversely shearable plate model incorporating the geometrical nonlinearities. The assumptions due to Ambartsumyan et al. [1,2] about the induced magnetic and electric intensity fields within the electroconductive plates are adopted to develop a 2-D electrodynamic model suitable for laminated composite plates. In order to capture the boundary layer effect on thermal conduction, a special treatment is presented in Section 4.3.

4.1. First-order transversely shearable plate model

According to the first-order transversely shearable plate theory [10,11], the following representation of the 3-D displacement field is postulated:

$$V_1(x_1, x_2, x_3, t) = v_1(x_1, x_2, t) + x_3\beta_1(x_1, x_2, t), \tag{12a}$$

$$V_2(x_1, x_2, x_3, t) = v_2(x_1, x_2, t) + x_3\beta_2(x_1, x_2, t), \tag{12b}$$

$$V_3(x_1, x_2, x_3, t) = v_3(x_1, x_2, t), \tag{12c}$$

in which β_1, β_2 are the rotations of the transverse normal of the plate mid-plane about the x_2- and the x_1-axis, respectively (see Fig. 2 for the definitions of their positive directions).

With the adoption of the von-Kármán strain concept, the 3-D Lagrangian strains are simplified as

$$\varepsilon_{11} = V_{1,1} + \tfrac{1}{2}(V_{3,1})^2 = \varepsilon_{11}^0 + x_3\varepsilon_{11}^1, \tag{13a}$$

$$\gamma_{12} = V_{1,2} + V_{2,1} + V_{3,1}V_{3,2} = \gamma_{12}^0 + x_3\gamma_{12}^1, \quad \gamma_{13} = V_{1,3} + V_{3,1} = \gamma_{13}^0, \tag{13b, c}$$

$$\varepsilon_{22} = V_{2,2} + \tfrac{1}{2}(V_{3,2})^2 = \varepsilon_{22}^0 + x_3\varepsilon_{22}^1, \quad \gamma_{23} = \gamma_{23}^0, \quad \varepsilon_{33} = 0, \tag{13d−f}$$

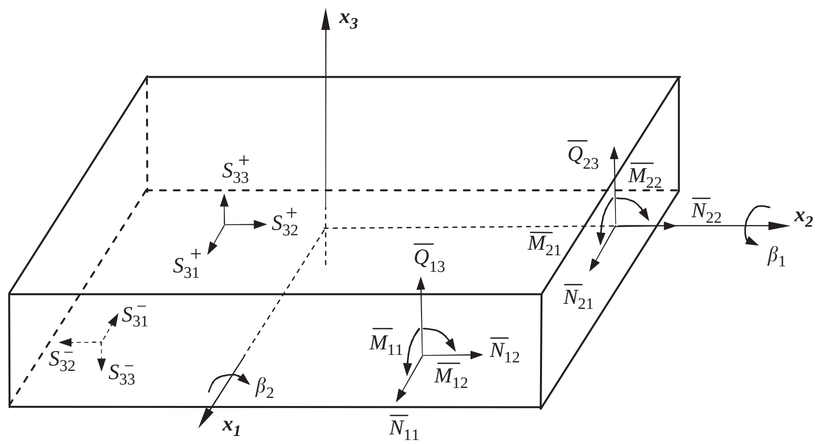

Fig. 2. Stress resultants and stress couples on a plate element.

where

$$\varepsilon_{11}^0 = v_{1,1} + \tfrac{1}{2}(v_{3,1})^2, \quad \varepsilon_{11}^1 = \beta_{1,1}, \quad \gamma_{12}^0 = v_{1,2} + v_{2,1} + v_{3,1}v_{3,2}, \tag{14a–c}$$

$$\gamma_{12}^1 = \beta_{1,2} + \beta_{2,1}, \quad \gamma_{13}^0 = \beta_1 + v_{3,1}, \quad \varepsilon_{22}^0 = v_{2,2} + \tfrac{1}{2}(v_{3,2})^2, \tag{14d–f}$$

$$\varepsilon_{22}^1 = \beta_{2,2}, \quad \gamma_{23}^0 = \beta_2 + v_{3,2}. \tag{14g, h}$$

Following the physical reasoning regarding the plate stress status (see e.g., Ref. [11, p. 126]), it is assumed $S_{33} = 0$. Use of this assumption in Eq. (5) and elimination of ε_{33} yield the following reduced constitutive equation:

$$\begin{Bmatrix} S_{11} \\ S_{22} \\ S_{12} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \left(\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \gamma_{12} \end{Bmatrix} - \begin{Bmatrix} \bar{\alpha}_{11} \\ \bar{\alpha}_{22} \\ \bar{\alpha}_{12} \end{Bmatrix} \Theta \right), \tag{15a}$$

$$\begin{Bmatrix} S_{23} \\ S_{13} \end{Bmatrix} = \begin{bmatrix} Q_{44} & Q_{45} \\ Q_{45} & Q_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{23} \\ \gamma_{13} \end{Bmatrix}, \tag{15b}$$

where $\bar{Q}_{ij}$ $(i,j = 1,2,6)$ and $\bar{\alpha}_{\alpha\beta}$ are the reduced elastic moduli and the reduced thermal coefficients, respectively.

The equations of motion of magneto-thermo-elastic plates and the consistent boundary conditions can be systematically derived via the extended Hamilton's principle, according to which:

$$\int_{t_1}^{t_2} (\delta\mathscr{T} - \delta\mathscr{U} + \overline{\delta W_e})\,\mathrm{d}t = 0, \tag{16a}$$

with

$$\delta v_i = 0, \quad \delta\beta_\alpha = 0 \quad \text{at } t = t_1 \text{ and } t_2, \tag{16b}$$

where $\delta\mathscr{T}$ and $\delta\mathscr{U}$ denote the virtual kinetic and strain energy, respectively, while $\overline{\delta W_e}$ denotes the virtual work due to external forces. For the problems to be addressed herein, these terms can be expressed as

$$\delta\mathscr{U} = \int_{\Omega_0} \int_{-h}^{h} [S_{11}\delta\varepsilon_{11} + S_{12}\delta\gamma_{12} + S_{13}\delta\gamma_{13} + S_{22}\delta\varepsilon_{22} + S_{23}\delta\gamma_{23}]\,\mathrm{d}\Omega_0\,\mathrm{d}x_3, \tag{17a}$$

$$\delta\mathscr{T} = \int_{\Omega_0} \int_{-h}^{h} \rho_0 [\dot{V}_1 \delta\dot{V}_1 + \dot{V}_2 \delta\dot{V}_2 + \dot{V}_3 \delta\dot{V}_3]\,\mathrm{d}\Omega_0\,\mathrm{d}x_3, \tag{17b}$$

$$
\begin{aligned}
\overline{\delta W_e} = & \int_{\Omega_0} \{[S_{33}^{+} - S_{33}^{-}]\delta v_3 + S_{31}^{+}[\delta v_1 + h\delta\beta_1] - S_{31}^{-}[\delta v_1 - h\delta\beta_1] \\
& + S_{32}^{+}[\delta v_2 + h\delta\beta_2] - S_{32}^{-}[\delta v_2 - h\delta\beta_2]\}\, \mathrm{d}\Omega_0 \\
& + \int_{\Omega_0}\int_{-h}^{h} \{f_1[\delta v_1 + x_3\delta\beta_1] + f_2[\delta v_2 + x_3\delta\beta_2] + f_3[\delta v_3]\}\, \mathrm{d}\Omega_0\, \mathrm{d}x_3 \\
& + \int_{-\ell_2}^{\ell_2} [\bar{N}_{11}\delta v_1 + \bar{N}_{12}\delta v_2 + \bar{N}_{13}\delta v_3 + \bar{M}_{11}\delta\beta_1 + \bar{M}_{12}\delta\beta_2]|_{-\ell_1}^{\ell_1}\, \mathrm{d}x_2 \\
& + \int_{-\ell_1}^{\ell_1} [\bar{N}_{12}\delta v_1 + \bar{N}_{22}\delta v_2 + \bar{N}_{23}\delta v_3 + \bar{M}_{12}\delta\beta_1 + \bar{M}_{22}\delta\beta_2]|_{-\ell_2}^{\ell_2}\, \mathrm{d}x_1. \qquad (17\text{c})
\end{aligned}
$$

In these equations, the domain Ω_0 refers to the undeformed plate mid-plane. $\bar{M}_{\alpha\beta}$, $\bar{N}_{\alpha\beta}$ and $\bar{N}_{\alpha 3}$ denote the applied edge forces, while S_{3i}^{+} and S_{3i}^{-} denote the applied surface tractions on the plate upper and bottom surfaces, respectively (see Fig. 2).

By defining the stress resultants and stress couples as

$$
N_{\alpha\beta} = N_{\beta\alpha} \equiv \int_{-h}^{h} S_{\alpha\beta}\, \mathrm{d}x_3, \quad N_{\alpha 3} \equiv \int_{-h}^{h} S_{\alpha 3}\, \mathrm{d}x_3, \qquad (18\text{a, b})
$$

$$
M_{\alpha\beta} = M_{\beta\alpha} \equiv \int_{-h}^{h} x_3 S_{\alpha\beta}\, \mathrm{d}x_3, \qquad (18\text{c})
$$

the equations of motion of the first-order transversely shearable plates can then be obtained:

$$
\delta v_1 : N_{11,1} + N_{12,2} - I_0\ddot{v}_1 + (S_{31}^{+} - S_{31}^{-}) + \underline{\int_{-h}^{h} f_1\, \mathrm{d}x_3} = 0, \qquad (19\text{a})
$$

$$
\delta v_2 : N_{12,1} + N_{22,2} - I_0\ddot{v}_2 + (S_{32}^{+} - S_{32}^{-}) + \underline{\int_{-h}^{h} f_2\, \mathrm{d}x_3} = 0, \qquad (19\text{b})
$$

$$
\begin{aligned}
\delta v_3 : & (N_{11}v_{3,1})_{,1} + (N_{12}v_{3,2})_{,1} + (N_{12}v_{3,1})_{,2} + N_{13,1} + (N_{22}v_{3,2})_{,2} + N_{23,2} \\
& - I_0\ddot{v}_3 + (S_{33}^{+} - S_{33}^{-}) + \underline{\int_{-h}^{h} f_3\, \mathrm{d}x_3} = 0, \qquad (19\text{c})
\end{aligned}
$$

$$
\delta\beta_1 : M_{11,1} + M_{12,2} - N_{13} - I_2\ddot{\beta}_1 + h(S_{31}^{+} + S_{31}^{-}) + \underline{\int_{-h}^{h} x_3 f_1\, \mathrm{d}x_3} = 0, \qquad (19\text{d})
$$

$$
\delta\beta_2 : M_{12,1} + M_{22,2} - N_{23} - I_2\ddot{\beta}_2 + h(S_{32}^{+} + S_{32}^{-}) + \underline{\int_{-h}^{h} x_3 f_2\, \mathrm{d}x_3} = 0. \qquad (19\text{e})
$$

In Eqs. (19a–e), the inertia terms I_0 and I_2 are defined as

$$(I_0, I_2) \equiv \int_{-h}^{h} (1, x_3^2)\rho_0 \, dx_3, \tag{20}$$

while the terms underscored by a solid line are of Lorentz-force origin.

The boundary conditions at $x_1 = \pm\ell_1$ are

$$\delta v_1 : (-N_{11} + \bar{N}_{11})\delta v_1 = 0, \quad \delta v_2 : (-N_{12} + \bar{N}_{12})\delta v_2 = 0, \tag{21a,b}$$

$$\delta v_3 : (-N_{11}v_{3,1} - N_{12}v_{3,2} - N_{13} + \bar{N}_{13})\delta v_3 = 0, \tag{21c}$$

$$\delta\beta_1 : (-M_{11} + \bar{M}_{11})\delta\beta_1 = 0, \quad \delta\beta_2 : (-M_{12} + \bar{M}_{12})\delta\beta_2 = 0, \tag{21d, e}$$

and at $x_2 = \pm\ell_2$,

$$\delta v_1 : (-N_{12} + \bar{N}_{12})\delta v_1 = 0, \quad \delta v_2 : \quad (-N_{22} + \bar{N}_{22})\delta v_2 = 0, \tag{22a, b}$$

$$\delta v_3 : (-N_{22}v_{3,2} - N_{12}v_{3,1} - N_{23} + \bar{N}_{23})\delta v_3 = 0, \tag{22c}$$

$$\delta\beta_1 : (-M_{12} + \bar{M}_{12})\delta\beta_1 = 0, \quad \delta\beta_2 : \quad (-M_{22} + \bar{M}_{22})\delta\beta_2 = 0. \tag{22d, e}$$

Based on their definitions (see Eqs. (18a–h)) and recalling the symmetry of the lay-up configuration, the stress resultants $N_{\alpha\beta}$, $N_{\alpha 3}$ and stress couples $M_{\alpha\beta}$ can be expressed, in terms of the Lagrangian strains, as

$$\begin{Bmatrix} N_{11} \\ N_{22} \\ N_{12} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{11}^0 - \tilde{\alpha}_{11}\Theta \\ \varepsilon_{22}^0 - \tilde{\alpha}_{22}\Theta \\ \gamma_{12}^0 - \tilde{\alpha}_{12}\Theta \end{Bmatrix}, \qquad \begin{Bmatrix} N_{23} \\ N_{13} \end{Bmatrix} = \begin{bmatrix} A_{44} & A_{45} \\ A_{45} & A_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{23}^0 \\ \gamma_{13}^0 \end{Bmatrix}, \tag{23a, b}$$

$$\begin{Bmatrix} M_{11} \\ M_{22} \\ M_{12} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{11}^1 \\ \varepsilon_{22}^2 \\ \gamma_{12}^1 \end{Bmatrix}. \tag{23c}$$

In Eqs. (23a–c),

$$(A_{ij}, D_{ij}) \equiv \int_{-h}^{h} (1, x_3^2)\bar{Q}_{ij} \, dx_3, \quad (i,j) = 1, 2, 6, \tag{24a}$$

while

$$\tilde{\alpha}_{\alpha\beta} \equiv \frac{\int_{-h}^{h} \bar{Q}_{\alpha\beta}\bar{\alpha}_{\alpha\beta} \, dx_3}{\int_{-h}^{h} \bar{Q}_{\alpha\beta} \, dx_3}, \quad \not\sum_{\alpha,\ \beta}, \tag{24b}$$

where the notation $\not\sum_{\alpha,\ \beta}$ indicates that there is no summation over α and β.

As a special case, the equations of motion for the non-shearable plate model can be obtained from Eqs. (19a–e). In such a case, Eqs. (19d) and (19e) are used to determine the transverse forces N_{13} and N_{23}, in place of the constitutive Eq. (23b). First, extract the transverse forces $N_{\alpha 3}$ from

Eqs. (19d,e) as

$$N_{13} = M_{11,1} + M_{12,2} - I_2\ddot{\beta}_1 + h(S_{31}^{+} + S_{31}^{-}) + \int_{-h}^{h} x_3 f_1 \, \mathrm{d}x_3, \tag{25a}$$

$$N_{23} = M_{12,1} + M_{22,2} - I_2\ddot{\beta}_2 + h(S_{32}^{+} + S_{32}^{-}) + \int_{-h}^{h} x_3 f_2 \, \mathrm{d}x_3, \tag{25b}$$

substituting these expressions in Eq. (19c), and then replacing β_1, β_2 by $-v_{3,1}$ and $-v_{3,2}$, respectively, gives as result that the equations of motion (19c–e) reduce to

$$\begin{aligned} &(N_{11}v_{3,1})_{,1} + (N_{12}v_{3,2})_{,1} + (N_{12}v_{3,1})_{,2} + (N_{22}v_{3,2})_{,2} + \Big[M_{11,11} + M_{12,12} + I_2\ddot{v}_{3,11} \\ &+ h(S_{31}^{+} + S_{31}^{-})_{,1} + \frac{\partial}{\partial x_1}\int_{-h}^{h} x_3 f_1 \, \mathrm{d}x_3\Big] + \Big[M_{12,12} + M_{22,22} + I_2\ddot{v}_{3,22} \\ &+ h(S_{32}^{+} + S_{32}^{-})_{,2} + \frac{\partial}{\partial x_2}\int_{-h}^{h} x_3 f_2 \, \mathrm{d}x_3\Big] - I_0\ddot{v}_3 + (S_{33}^{+} - S_{33}^{-}) + \int_{-h}^{h} f_3 \, \mathrm{d}x_3 = 0, \end{aligned} \tag{25c}$$

the remaining Eqs. (19a,b) being invariant for this case.

It is noted that in the constitutive equations (23a–c), β_1 and β_2 should also be replaced by $-v_{3,1}$, $-v_{3,2}$, respectively.

4.2. Electrodynamics

As it was previously mentioned, within the present plate model only geometrical nonlinearities will be retained. Consequently, due to the implementation of the small disturbance concept, these equations will be rendered linear from the electromagnetic point of view. To this end, the electromagnetic field quantities are represented as

$$\mathbf{E} = \mathbf{E}_0 + \mathbf{e}, \quad \mathbf{H} = \mathbf{H}_0 + \mathbf{h}, \quad \mathbf{B} = \mathbf{B}_0 + \mathbf{b}. \tag{26a$-$c}$$

In these equations, $\mathbf{e}(\equiv \mathbf{e}(x_1, x_2, x_3, t))$, $\mathbf{h}(\equiv \mathbf{h}(x_1, x_2, x_3, t))$ and $\mathbf{b}(\equiv \mathbf{b}(x_1, x_2, x_3, t))$ denote the small disturbances of the primary electromagnetic field quantities, $\mathbf{E}_0$, $\mathbf{H}_0$ and $\mathbf{B}_0$, respectively. As a result, the square of disturbance quantities are second-order terms that are negligibly small when compared with the undisturbed primary electromagnetic field quantities. Such a procedure of linearization will be applied next.

For the problem at hand, $\mathbf{E}_0$ is a zero quantity, and only the induced electric field vector is different from zero, implying that for this case, Eq. (26a) should be replaced by $\mathbf{E} = \mathbf{0} + \mathbf{e}$. The Maxwell's equations for the disturbed electromagnetic field become

$$\operatorname{curl}\mathbf{e} = -\dot{\mathbf{b}}, \quad \operatorname{curl}\mathbf{h} = \mathbf{j}, \quad \mathbf{j} = \mathbb{G}(\mathbf{e} + \dot{\mathbf{V}} \times \mathbf{B}_0). \tag{27a$-$c}$$

By adopting in our approach the assumptions due to Ambartsumyan et al. [1,2] about the through-thickness variation of the components of $\mathbf{e}(x_1, x_2, x_3, t)$ and $\mathbf{h}(x_1, x_2, x_3, t)$, one postulates:

$$e_1 \equiv e_1(x_1, x_2, t) \equiv \varphi(x_1, x_2, t), \quad e_2 \equiv e_2(x_1, x_2, t) \equiv \psi(x_1, x_2, t), \tag{28a, b}$$

$$h_3 \equiv h_3(x_1, x_2, t) \equiv \chi(x_1, x_2, t). \tag{28c}$$

In other words, in accordance with the model in Refs. [1,2], which was derived via the asymptotic analysis of 3-D magnetoelastic field equations, the tangential components of $\mathbf{e}$ (i.e. e_1 and e_2) and the transversal components of $\mathbf{h}$ (i.e., h_3) are uniform across the plate thickness. Once functions φ, ψ and χ are determined, the remaining components, i.e., e_3, h_1 and h_2, can be determined from the relevant electrodynamic equations.

In the following, we will represent the undisturbed magnetic field within the plate in the form $\mathbf{B}_0 = \mathbf{B}_0^0 + x_3\mathbf{B}_0^1$, where $\mathbf{B}_0^0$ and $\mathbf{B}_0^1$ are constant vectors.

4.2.1. Determination of the outer magnetic field

In conjunction with Eq. (28c), the jump condition (8b) reduces to

$$h_3^{(e)} = \chi(x_1, x_2, t). \tag{29}$$

By defining the potential $\Phi^{(e)}$, such that $\nabla\Phi^{(e)} = \mathbf{h}^{(e)}$, the problem of determining the outer magnetic field reduces to

$$\nabla^2\Phi^{(e)} = 0, \tag{30a}$$

$$\left.\frac{\partial\Phi^{(e)}}{\partial x_3}\right|_{x_3=\pm h} = \chi(x_1, x_2, t), \quad |x_1| < \ell_1, \quad |x_2|\ell_2, \tag{30b}$$

$$|\nabla\Phi^{(e)}| \to 0, \quad \text{as } x_1^2 + x_2^2 + x_3^2 \to \infty. \tag{30c}$$

It is noted that the boundary conditions at the edge surfaces ($|x_1| = \ell_1$, $|x_3| \leqslant h$, $|x_2| \leqslant \ell_2$) and ($|x_2| = \ell_2$, $|x_3| \leqslant h$, $|x_1| \leqslant \ell_1$) will be disregarded in the following asymptotic approach.

Expand the left-hand side of the boundary condition (30b) in a Taylor series along the x_3 direction as

$$\frac{\partial}{\partial x_3}\Phi^{(e)}(x_1, x_2, \pm h, t) = \frac{\partial}{\partial x_3}\Phi^{(e)}(x_1, x_2, 0, t) \pm h\frac{\partial^2}{\partial x_3^2}\Phi^{(e)}(x_1, x_2, 0, t) + \mathcal{O}(\varepsilon^2), \tag{31}$$

in which $\varepsilon(\equiv h/\min(\ell_1, \ell_2))$ is a small parameter. Since $\Phi^{(e)}$ is defined as the potential of the induced magnetic field, it can be expanded asymptotically in the following form:

$$\Phi^{(e)} = \Phi_1^{(e)} + \Phi_2^{(e)} + \cdots, \tag{32}$$

in which $\Phi_i^{(e)} \sim \mathcal{O}(\varepsilon^i)$.

Substituting expressions (31) and (32) into Eqs. (30a–c), the asymptotic representation of the problem can be obtained as follows:

Order $\mathcal{O}(\varepsilon)$:

$$\nabla^2\Phi_1^{(e)} = 0, \tag{33a}$$

$$\left.\frac{\partial\Phi_1^{(e)}}{\partial x_3}\right|_{x_3=0\pm} = \chi(x_1, x_2, t) \quad |x_1| < \ell_1, \quad |x_2| < \ell_2, \tag{33b}$$

$$|\nabla\Phi_1^{(e)}| \to 0, \quad \text{as } x_1^2 + x_2^2 + x_3^2 \to \infty, \tag{33c}$$

Order $\mathcal{O}(\varepsilon^2)$:

$$\nabla^2 \Phi_2^{(e)} = 0, \tag{34a}$$

$$\left.\frac{\partial \Phi_2^{(e)}}{\partial x_3}\right|_{x_3=0\pm} = \mp h \left.\frac{\partial^2 \Phi_1^{(e)}}{\partial x_3^2}\right|_{x_3=0\pm}, \quad |x_1| < \ell_1, \quad |x_2| < \ell_2, \tag{34b}$$

$$|\nabla \Phi_2^{(e)}| \to 0, \quad \text{as } x_1^2 + x_2^2 + x_3^2 \to \infty. \tag{34c}$$

The first-order problem (i.e., $\mathcal{O}(\varepsilon)$) represented by Eqs. (33a–c) is an *antisymmetric* one due to the antisymmetry of the corresponding solution with respect to x_3, while Eqs. (34a–c) describe the thickness effect of the plate on the induced magnetic field. It is clearly seen that compared with the antisymmetric problem, the thickness influence represents a higher order correction and will be neglected in this article.

Following the solution procedures usually adopted in aerodynamics [19], Eqs. (33a–c) can be solved by distributing vortices on the plate boundaries as to fulfill the boundary conditions (33b). It is recalled that the vortices automatically fulfill the Laplace equation (33a) and the far-field condition (33c) (see Ref. [19, pp. 69–71]).

Denoting the x_1 and x_2 components of the vortices as γ_2 and γ_1, respectively, it can readily be shown that

$$h_1^{\pm} \equiv h_1(x_1, x_2, 0\pm, t) = \left.\frac{\partial \Phi_1^{(e)}}{\partial x_1}\right|_{x_3=0\pm} = \pm \frac{\gamma_2}{2}, \tag{35a}$$

$$h_2^{\pm} \equiv h_2(x_1, x_2, 0\pm, t) = \left.\frac{\partial \Phi_1^{(e)}}{\partial x_2}\right|_{x_3=0\pm} = \mp \frac{\gamma_1}{2}, \tag{35b}$$

$$-\gamma_{1,1} = \gamma_{2,2}. \tag{35c}$$

Based on the Biot–Savart law (see e.g., Ref. [20]),

$$\mathrm{d}\mathbf{h}^{(e)} = \frac{[\gamma_1 \mathbf{i}_1 + \gamma_2 \mathbf{i}_2]\,\mathrm{d}s_1\,\mathrm{d}s_2 \times [(x_1 - s_1)\mathbf{i}_1 + (x_2 - s_2)\mathbf{i}_2 + x_3 \mathbf{i}_3]}{4\pi[(x_1 - s_1)^2 + (x_2 - s_2)^2 + x_3^2]^3}, \tag{36}$$

and integrating Eq. (36) over the undeformed mid-plane, the solutions of $h_1^{(e)}$ and $h_2^{(e)}$ can be represented as

$$h_1^{(e)}(x_1, x_2, x_3, t) = \frac{x_3}{4\pi} \int_{\Omega_0} \frac{\gamma_2(s_1, s_2, t)\,\mathrm{d}s_1 s_2}{[(x_1 - s_1)^2 + (x_2 - s_2)^2 + x_3^2]^{3/2}}, \tag{37a}$$

$$h_2^{(e)}(x_1, x_2, x_3, t) = \frac{x_3}{4\pi} \int_{\Omega_0} \frac{-\gamma_1(s_1, s_2, t)\,\mathrm{d}s_1 s_2}{[(x_1 - s_1)^2 + (x_2 - s_2)^2 + x_3^2]^{3/2}}, \tag{37b}$$

$$h_3^{(e)}(x_1, x_2, x_3, t) = \frac{1}{4\pi} \int_{\Omega_0} \frac{\gamma_1(s_1, s_2, t)[x_2 - s_2] - \gamma_2(s_1, s_2, t)[x_1 - s_1]}{[(x_1 - s_1)^2 + (x_2 - s_2)^2 + x_3^2]^{3/2}}\,\mathrm{d}s_1 s_2. \tag{37c}$$

Enforcing the boundary condition (33b), the following relation is reached:

$$\chi(x_1,x_2,t)=\frac{1}{4\pi}\int_{\Omega_0}\frac{\gamma_1(s_1,s_2,t)[x_2-s_2]-\gamma_2(s_1,s_2,t)[x_1-s_1]}{[(x_1-s_1)^2+(x_2-s_2)^2]^{3/2}}\,\mathrm{d}s_1s_2. \tag{38}$$

Next, considering the physical restriction related to the absence of jumps for the magnetic potential $\Phi_1^{(e)}$ at the boundaries of the plate [21], i.e.,

$$\Phi_1^{(e)}(x_1,x_2,0^+,t)-\Phi_1^{(e)}(x_1,x_2,0^-,t)=0,\quad x_1=\pm\ell_1,\ \text{or}\ x_2=\pm\ell_2, \tag{39}$$

it then follows that

$$\int_{-\ell_2}^{\ell_2}\gamma_1(s_1,s_2,t)\,\mathrm{d}s_2=0,\qquad \int_{-\ell_1}^{\ell_1}\gamma_2(s_1,s_2,t)\,\mathrm{d}s_1=0. \tag{40a,b}$$

By using relation (35c) and the boundary conditions (40), Eq. (38) can be significantly simplified. The final result can be expressed as

$$\chi(x_1,x_2,t)=-\frac{1}{4\pi}\int_{\Omega_0}\frac{\partial\gamma_2(s_1,s_2,t)}{\partial s_2}\frac{\sqrt{(x_1-s_1)^2+(x_2-s_2)^2}}{(x_1-s_1)(x_2-s_2)}\,\mathrm{d}s_1\,\mathrm{d}s_2. \tag{41}$$

Based on Eq. (41) and the boundary conditions (40b), γ_2 can be determined in terms of χ. Then Eq. (35c) in conjunction with the boundary conditions (40a) enables one to determine γ_1.

We note that Eq. (41) is valid for rectangular plates. Two special cases are further remarked on:

- For infinite plates (i.e., $\ell_1\to\infty$, $\ell_2\to\infty$), Eq. (41) can be solved, at least theoretically, via the double Fourier transform technique. However, for a more convenient approach, the source distribution method is used to directly express γ_1 in term of χ. Such an approach was carried out in Ref. [22].
- For plate strips (e.g., $\ell_2\to\infty$), $\gamma_1=0$ and γ_2 is independent of x_2. In such a case, from the 2-D potential theory, the counterpart of Eq. (41) reduces to

$$\chi(x_1,t)=\frac{1}{2\pi}\int_{-\ell_1}^{\ell_1}\frac{\gamma_2(s_1,t)}{s_1-x_1}\,\mathrm{d}s_1. \tag{42}$$

4.2.2. Electrodynamics within the plate

Based on representations (28a–c), Ampère's law (27b), can be expressed as

$$h_{2,3}=\chi_{,2}-g_{11}[\varphi+\dot V_2B_{03}-\dot V_3B_{02}]-g_{12}[\psi+\dot V_3B_{01}-\dot V_1B_{03}], \tag{43a}$$

$$h_{1,3}=\chi_{,1}+g_{12}[\varphi+\dot V_2B_{03}-\dot V_3B_{02}]-g_{22}[\psi+\dot V_3B_{01}-\dot V_1B_{03}], \tag{43b}$$

$$h_{2,1}-h_{1,2}=g_{33}[e_3+\dot V_1B_{03}-\dot V_2B_{01}]. \tag{43c}$$

In the above equations, V_i are the displacement components defined in Eqs. (12a–c). Faraday's law (27a) can be represented as

$$e_{3,2}=-\mu_0\dot h_1,\quad e_{3,1}=\mu_0\dot h_2,\quad \psi_{,1}-\varphi_{,2}=-\mu_0\dot\chi. \tag{44a–c}$$

As it will be shown later, in conjunction with the solution of the outside magnetic field $\mathbf{h}^e$, Eqs. (43a,b) and (44c) will be used to determine the unknowns φ, ψ and χ, and once h_1 and h_2 are determined, Eq. (43c) can be used to determine e_3. It is noted that based on the assumptions (28a–c), Eqs. (44a, b) may not be exactly fulfilled.

Integrating Eq. (43a) across the thickness of the ith layer, we get

$$\begin{aligned}\int_{z_i}^{z_{i+1}} h_{2,3}\,\mathrm{d}x_3 &= h_2^{(i+1)} - h_2^{(i)} \\ &= \Xi h\Big\{ \chi_{,2} - g_{11}^{(i)}(\varphi + \dot{v}_2 B_{03}^0 - \dot{v}_3 B_{02}^0) - g_{12}^{(i)}(\psi + \dot{v}_3 B_{01}^0 - \dot{v}_1 B_{03}^0) \\ &\quad - g_{11}^{(i)}(\dot{v}_2 B_{03}^1 + \dot{\beta}_2 B_{03}^0 - \dot{v}_3 B_{02}^1)\left(\frac{z_{i+1}+z_i}{2}\right) - g_{12}^{(i)}(\dot{v}_3 B_{01}^1 - \dot{\beta}_1 B_{03}^0 - \dot{v}_1 B_{03}^1)\left(\frac{z_{i+1}+z_i}{2}\right) \\ &\quad - [g_{11}^{(i)}\dot{\beta}_2 + g_{12}^{(i)}\dot{\beta}_1]\left(\frac{z_{i+1}^2 + z_{i+1}z_i + z_i}{3}\right)B_{03}^1 \Big\} \triangleq \tilde{a}_i, \quad i = \overline{1,\ N_L}. \end{aligned} \tag{45}$$

As it can be obtained from Eq. (35b), summation of Eq. (45) throughout all the constituent layers (implying that i goes from 1 to N_L) and assuming continuity of h_i across the contiguous layer interfaces lead to the following relation:

$$\sum_{i=1}^{N_L}\int_{z_i}^{z_{i+1}} h_{2,3}\,\mathrm{d}x_3 = h_2^{(N_L+1)} - h_2^{(1)} = h_2^+ - h_2^- = -\gamma_1(x_1, x_2, t), \tag{46}$$

where $h_2^{(N_L+1)} = h_2^+$ and $h_2^{(1)} = h_2^-$ are derived from Eq. (35b).

Starting from Eq. (43b) and applying a similar procedure to $h_{2,3}$, in conjunction with Eq. (35a), another relation involving φ, ψ and χ can be derived as

$$\sum_{i=1}^{N_L}\int_{z_i}^{z_{i+1}} h_{1,3}\,\mathrm{d}x_3 = h_1^{(N_L+1)} - h_1^{(1)} = h_1^+ - h_1^- = \gamma_2(x_1, x_2, t). \tag{47}$$

In short, the governing equations for the unknown φ, ψ and χ, are

$$\chi_{,2} - \mathscr{M}_0[g_{11}]\varphi - \mathscr{M}_0[g_{12}]\psi = -\frac{\gamma_1}{2h} + \Gamma_1, \tag{48a}$$

$$\chi_{,1} + \mathscr{M}_0[g_{12}]\varphi + \mathscr{M}_0[g_{22}]\psi = \frac{\gamma_2}{2h} - \Gamma_2, \tag{48b}$$

$$\psi_{,1} - \varphi_{,2} = -\mu_0\dot{\chi}, \tag{48c}$$

in which, Γ_α are defined in Appendix A, and the operator $\mathscr{M}_0[g_{\alpha\beta}] \equiv 1/N_L\sum_{i=1}^{N_L} g_{\alpha\beta}^{(i)}$.

Expressing in Eqs. (48a, b) φ and ψ in terms of χ, v_i and β_α, and substituting the results into Eq. (48c), the following diffusion equation associated with χ can be derived:

$$\begin{aligned}&\mathscr{M}_0[g_{11}]\chi_{,11} + 2\mathscr{M}_0[g_{12}]\chi_{,12} + \mathscr{M}_0[g_{22}]\chi_{,22} - \mu_0\bar{\Xi}\dot{\chi} \\ &\quad = \mathscr{M}_0[g_{12}]\left[-\frac{1}{h}\gamma_{1,1} + \Gamma_{1,1} - \Gamma_{2,2}\right] \\ &\quad + \mathscr{M}_0[g_{11}]\left[\frac{1}{2h}\gamma_{2,1} - \Gamma_{2,1}\right] + \mathscr{M}_0[g_{22}]\left[-\frac{1}{2h}\gamma_{1,2} + \Gamma_{1,2}\right],\end{aligned} \tag{49}$$

where $\bar{\Xi} \equiv (\mathscr{M}_0[g_{11}]\mathscr{M}_0[g_{22}] - \mathscr{M}_0[g_{12}]^2)$. We note that for deriving Eq. (49), Eq. (35c) has been used.

Given the boundary and initial conditions for χ, based on Eq. (49), the unknown χ can be uniquely determined. Then the solutions for φ and ψ become

$$\begin{Bmatrix} \varphi \\ \psi \end{Bmatrix} = \frac{1}{\bar{\Xi}} \begin{Bmatrix} \mathscr{M}_0[g_{22}]\left[\chi_{,2} + \frac{\gamma_1}{2h} - \Gamma_1\right] + \mathscr{M}_0[g_{12}]\left[\chi_{,1} - \frac{\gamma_2}{2h} + \Gamma_2\right] \\ -\mathscr{M}_0[g_{12}]\left[\chi_{,2} + \frac{\gamma_1}{2h} - \Gamma_1\right] - \mathscr{M}_0[g_{11}]\left[\chi_{,1} - \frac{\gamma_2}{2h} + \Gamma_2\right] \end{Bmatrix}. \tag{50}$$

Once χ, φ and ψ are obtained, in conjunction with the boundary conditions (35a,b), from Eqs. (43a–c), h_1, h_2 and e_3 can be determined. Their expressions are

$$\begin{aligned} h_2(x_1,x_2,x_3,t) &= h_2(x_1,x_2,z_i,t) \\ &+ (x_3 - z_i)\{\chi_{,2} - g_{11}^{(i)}[\varphi + \dot{v}_2 B_{03}^0 - \dot{v}_3 B_{02}^0] - g_{12}^{(i)}[\psi + \dot{v}_3 B_{01}^0 - \dot{v}_1 B_{03}^0]\} \\ &- \left(\frac{x_3^2 - z_i^2}{2}\right)\{g_{11}^{(i)}[\dot{v}_2 B_{03}^1 + \dot{\beta}_2 B_{03}^0 - \dot{v}_3 B_{02}^1] + g_{12}^{(i)}[\dot{v}_3 B_{01}^1 - \dot{\beta}_1 B_{03}^0 - \dot{v}_1 B_{03}^1]\} \\ &- \left(\frac{x_3^3 - z_i^3}{3}\right)[g_{11}^{(i)}\dot{\beta}_2 + g_{12}^{(i)}\dot{\beta}_1]B_{03}^1, \quad x_3 \in [z_i, z_{i+1}), \end{aligned} \tag{51a}$$

$$\begin{aligned} h_1(x_1,x_2,x_3,t) &= h_1(x_1,x_2,z_i,t) \\ &+ (x_3 - z_i)\{\chi_{,1} + g_{12}^{(i)}[\varphi + \dot{v}_2 B_{03}^0 - \dot{v}_3 B_{02}^0] - g_{22}^{(i)}[\psi + \dot{v}_3 B_{01}^0 - \dot{v}_1 B_{03}^0]\} \\ &+ \left(\frac{x_3^2 - z_i^2}{2}\right)\{g_{12}^{(i)}[\dot{v}_2 B_{03}^1 + \dot{\beta}_2 B_{03}^0 - \dot{v}_3 B_{02}^1] + g_{22}^{(i)}[\dot{v}_3 B_{01}^1 - \dot{\beta}_1 B_{03}^0 - \dot{v}_1 B_{03}^1]\} \\ &+ \left(\frac{x_3^3 - z_i^3}{3}\right)[g_{12}^{(i)}\dot{\beta}_2 - g_{22}^{(i)}\dot{\beta}_1]B_{03}^1, \quad x_3 \in [z_i, z_{i+1}), \end{aligned} \tag{51b}$$

$$\begin{aligned} e_3(x_1,x_2,x_3,t) &= \frac{1}{g_{33}}[h_{2,1} - h_{1,2}] + [-\dot{v}_1 B_{03}^0 + \dot{v}_2 B_{01}^0] \\ &+ x_3[\dot{v}_2 B_{01}^1 + \dot{\beta}_2 B_{01}^0 - \dot{v}_1 B_{03}^1 - \dot{\beta}_1 B_{03}^0] + x_3^2[\dot{\beta}_2 B_{01}^1 - \dot{\beta}_1 B_{03}^1], \quad x_3 \in [z_i, z_{i+1}). \end{aligned} \tag{51c}$$

Based on the recurrence relation (45), $h_2(x_1,x_2,z_k,t) \equiv h_2^{(k)}$ $(k = \overline{2, N_L})$ can be successively determined, starting from $h_2(x_1,x_2,z_1,t) = -\gamma_1/2$. A similar procedure is applicable to $h_1^{(k)}$. We note that the solutions of $h_1^{(k)}$ and $h_2^{(k)}$ can also be directly derived. In a closed form, using Eq. (45) and the fact that $h_2^{(1)} = -h_2^{(N_L+1)}$, $h_1^{(k)}$ and $h_2^{(k)}$ are obtained as

$$\begin{Bmatrix} h_2^{(1)} \\ h_2^{(2)} \\ \vdots \\ h_2^{(N_L)} \end{Bmatrix} = \begin{bmatrix} -1/2 & -1/2 & \cdots & -1/2 \\ 1/2 & -1/2 & \cdots & -1/2 \\ \vdots & \vdots & \vdots & \vdots \\ 1/2 & 1/2 & \cdots & -1/2 \end{bmatrix} \begin{Bmatrix} \tilde{a}_1 \\ \tilde{a}_2 \\ \vdots \\ \tilde{a}_{N_L} \end{Bmatrix}, \tag{52}$$

where $\tilde{a}_k$ $(k = \overline{1, N_L})$ are defined in Eq. (45). The solution of $h_1^{(k)}$ $(k = 1, N_L)$ can be derived similarly.

The solutions for the induced current j_1, j_2 and j_3 as defined by Eq. (27b) are listed in Appendix A.

4.3. Thermal equations

Given the thermal boundary condition (11), we postulate that the distribution of the thermal field within the plate can be represented as

$$\Theta(x_1, x_2, x_3, t) = \Theta_0(x_1, x_2, t) + \frac{x_3}{h}\,\Theta_1(x_1, x_2, t), \tag{53}$$

in which

$$\Theta_0(x_1, x_2, t) = \frac{1}{2h}\int_{-h}^{h} \Theta(x_1, x_2, x_3, t)\,\mathrm{d}x_3, \tag{54a}$$

$$\Theta_1(x_1, x_2, t) = \frac{3}{2h^2}\int_{-h}^{h} x_3\Theta(x_1, x_2, x_3, t)\,\mathrm{d}x_3. \tag{54b}$$

Starting with the boundary conditions (11), we get the following relations:

$$k_{33}^{(N_L)}\frac{\partial\Theta}{\partial x_3}\bigg|_{x_3=h} = -\alpha_s^+(\Theta_0 + \Theta_1 - \Theta_s^+), \tag{55a}$$

$$k_{33}^{(1)}\frac{\partial\Theta}{\partial x_3}\bigg|_{x_3=-h} = \alpha_s^-(\Theta_0 - \Theta_1 - \Theta_s^-). \tag{55b}$$

Before dealing with the 3-D heat conduction equation, Eq. (1f), we assume that there is a perfect bonding of the contiguous laminae and there is no generated or absorbed heat on the interfaces of the laminae, implying the validity of the continuity condition of heat flux across the interfaces:

$$k_{33}^{(i)}\frac{\partial\Theta}{\partial x_3}\bigg|_{z_{i+1}} = k_{33}^{(i+1)}\frac{\partial\Theta}{\partial x_3}\bigg|_{z_{i+1}} \quad (i = \overline{2, N_L - 1}). \tag{56}$$

In order to enforce boundary conditions (55a,b), it is not proper to directly substitute Eq. (53) into Eq. (1f). Instead, by invoking condition (56), it can be proved that

$$\begin{aligned}\sum_{i=1}^{N_L}\int_{z_i}^{z_{i+1}} k_{33}^{(i)}\frac{\partial^2\Theta}{\partial x_3^2}\,\mathrm{d}x_3 &= k_{33}^{(N_L)}\frac{\partial\Theta}{\partial x_3}\bigg|_{x_3=h} - k_{33}^{(1)}\frac{\partial\Theta}{\partial x_3}\bigg|_{x_3=-h}\\ &= -(\alpha_s^+ + \alpha_s^-)\Theta_0 - (\alpha_s^+ - \alpha_s^-)\Theta_1 + (\alpha_s^+\Theta_s^+ + \alpha_s^-\Theta_s^-),\end{aligned} \tag{57a}$$

$$\begin{aligned}\sum_{i=1}^{N_L}\int_{z_i}^{z_{i+1}} x_3 k_{33}^{(i)}\frac{\partial^2\Theta}{\partial x_3^2}\,\mathrm{d}x_3 &= h\left[k_{33}^{(N_L)}\frac{\partial\Theta}{\partial x_3}\bigg|_{x_3=h} + k_{33}^{(1)}\frac{\partial\Theta}{\partial x_3}\bigg|_{x_3=-h}\right] - 2\Theta_1\mathscr{M}_0[k_{33}]\\ &= h[-(\alpha_s^+ - \alpha_s^-)\Theta_0 - (\alpha_s^+ + \alpha_s^-)\Theta_1 + (\alpha_s^+\Theta_s^+ - \alpha_s^-\Theta_s^-)] - 2\Theta_1\mathscr{M}_0[k_{33}].\end{aligned} \tag{57b}$$

Now, taking the moments of order zero and one of Eq. (1f) in the sense of

$$\frac{1}{2h}\sum_{i=1}^{N_L}\int_{z_i}^{z_{i+1}}(\text{Eq. (1f)})\,\mathrm{d}x_3,\quad \frac{3}{2h^2}\sum_{i=1}^{N_L}\int_{z_i}^{z_{i+1}}x_3\,(\text{Eq. (1f)})\,\mathrm{d}x_3, \tag{58a, b}$$

we get the governing equations for Θ_0 and Θ_1:

$$\begin{aligned}
&\mathscr{M}_0[k_{11}]\Theta_{0,11}+\frac{1}{h}\mathscr{M}_1[k_{11}]\Theta_{1,11}+2\mathscr{M}_0[k_{12}]\Theta_{0,12}+\frac{2}{h}\mathscr{M}_1[k_{12}]\Theta_{1,12}+\mathscr{M}_0[k_{22}]\Theta_{0,22}\\
&\quad+\frac{1}{h}\mathscr{M}_1[k_{22}]\Theta_{1,22}-\frac{1}{2h}[(\alpha_s^++\alpha_s^-)\Theta_0+(\alpha_s^+-\alpha_s^-)\Theta_1-(\alpha_s^+\Theta_s^++\alpha_s^-\Theta_s^-)]\\
&\quad-\mathscr{M}_0[C_t]\dot{\Theta}_0-\frac{1}{h}\mathscr{M}_1[C_t]\dot{\Theta}_1+\frac{1}{N_L}\sum_{i=1}^{N_L}\left[\frac{g_{22}^{(i)}}{\Xi^{(i)}}J_{01}^2-2\frac{g_{12}^{(i)}}{\Xi^{(i)}}J_{01}J_{02}+\frac{g_{11}^{(i)}}{\Xi^{(i)}}J_{02}^2\right]\\
&\quad+\sum_{i=1}^{N_L}\left[\frac{g_{22}^{(i)}}{\Xi^{(i)}}J_{01}-\frac{g_{12}^{(i)}}{\Xi^{(i)}}J_{02}\right]\frac{1}{h}\int_{z_i}^{z_{i+1}}j_1\,\mathrm{d}x_3+\sum_{i=1}^{N_L}\left[\frac{g_{11}^{(i)}}{\Xi^{(i)}}J_{02}-\frac{g_{12}^{(i)}}{\Xi^{(i)}}J_{01}\right]\frac{1}{h}\int_{z_i}^{z_{i+1}}j_2\,\mathrm{d}x_3=0,
\end{aligned} \tag{59a}$$

$$\begin{aligned}
&\frac{1}{h}\mathscr{M}_1[k_{11}]\Theta_{0,11}+\frac{1}{h^2}\mathscr{M}_2[k_{11}]\Theta_{1,11}+\frac{2}{h}\mathscr{M}_1[k_{12}]\Theta_{0,12}+\frac{2}{h^2}\mathscr{M}_2[k_{12}]\Theta_{1,12}\\
&\quad+\frac{1}{h}\mathscr{M}_1[k_{22}]\Theta_{0,22}+\frac{1}{h^2}\mathscr{M}_2[k_{22}]\Theta_{1,22}-\frac{1}{2h}\big[(\alpha_s^+-\alpha_s^-)\Theta_0+(\alpha_s^++\alpha_s^-)\Theta_1\\
&\quad-\alpha_s^+\Theta_s^++\alpha_s^-\Theta_s^-\big]-\frac{1}{h^2}\Theta_1\mathscr{M}_0[k_{33}]-\frac{1}{h}\mathscr{M}_1[C_t]\dot{\Theta}_0-\frac{1}{h^2}\mathscr{M}_2[C_t]\dot{\Theta}_1\\
&\quad+\frac{1}{N_L}\sum_{i=1}^{N_L}\frac{(z_{i+1}+z_i)}{2h}\left[\frac{g_{22}^{(i)}}{\Xi^{(i)}}J_{01}^2-2\frac{g_{12}^{(i)}}{\Xi^{(i)}}J_{01}J_{02}+\frac{g_{11}^{(i)}}{\Xi^{(i)}}J_{02}^2\right]\\
&\quad+\sum_{i=1}^{N_L}\left[\frac{g_{22}^{(i)}}{\Xi^{(i)}}J_{01}-\frac{g_{12}^{(i)}}{\Xi^{(i)}}J_{02}\right]\frac{1}{h^2}\int_{z_i}^{z_{i+1}}x_3j_1\,\mathrm{d}x_3\\
&\quad+\sum_{i=1}^{N_L}\left[\frac{g_{11}^{(i)}}{\Xi^{(i)}}J_{02}-\frac{g_{12}^{(i)}}{\Xi^{(i)}}J_{01}\right]\frac{1}{h^2}\int_{z_i}^{z_{i+1}}x_3j_2\,\mathrm{d}x_3=0,
\end{aligned} \tag{59b}$$

in which, $\Xi^{(i)}\equiv g_{11}^{(i)}g_{22}^{(i)}-[g_{12}^{(i)}]^2$ while j_1, j_2 and j_3 are defined in Appendix A.

4.4. Expressions of the 2-D generalized ponderomotive forces

The Lorentz forces in Eqs. (19a–e) can be expressed as

$$f_1=J_{02}B_{03}+J_{02}b_3+(j_2B_{03}-j_3B_{02}), \tag{60a}$$

$$f_2=-J_{01}B_{03}-J_{01}b_3+(j_3B_{01}-j_1B_{03}), \tag{60b}$$

$$f_3=(J_{01}B_{02}-J_{02}B_{01})+(J_{01}b_2-J_{02}b_1)+(j_1B_{02}-j_2B_{01}), \tag{60c}$$

where $b_1=\mu_0h_1$, $b_2=\mu_0h_2$, $b_3=\mu_0\chi$, while j_1, j_2 and j_3 are defined in Appendix A.

The 2-D generalized Lorentz forces in Eqs. (19a–e) can then be represented as

$$\int_{-h}^{h} f_1 \, dx_3 = 2hJ_{02}B_{03}^0 + 2hJ_{02}\chi + B_{03}^0 \int_{-h}^{h} j_2 \, dx_3 + B_{03}^1 \int_{-h}^{h} x_3 j_2 \, dx_3 - B_{02}^0 \int_{-h}^{h} j_3 \, dx_3 - B_{02}^1 \int_{-h}^{h} x_3 j_3 \, dx_3, \tag{61a}$$

$$\int_{-h}^{h} f_2 \, dx_3 = -2hJ_{01}B_{03}^0 - 2h\mu_0 J_{01}\chi + B_{01}^0 \int_{-h}^{h} j_3 \, dx_3 - B_{03}^0 \int_{-h}^{h} j_1 \, dx_3 + B_{01}^1 \int_{-h}^{h} x_3 j_3 \, dx_3 - B_{03}^1 \int_{-h}^{h} x_3 j_1 \, dx_3, \tag{61b}$$

$$\int_{-h}^{h} f_3 \, dx_3 = 2h[J_{01}B_{02}^0 - J_{02}B_{01}^0] + \mu_0 \left[J_{01} \int_{-h}^{h} h_2 \, dx_3 - J_{02} \int_{-h}^{h} h_1 \, dx_3 \right] + B_{02}^0 \int_{-h}^{h} j_1 \, dx_3 - B_{01}^0 \int_{-h}^{h} j_2 \, dx_3 + B_{02}^1 \int_{-h}^{h} x_3 j_1 \, dx_3 - B_{01}^1 \int_{-h}^{h} x_3 j_2 \, dx_3, \tag{61c}$$

$$\int_{-h}^{h} x_3 f_1 \, dx_3 = \frac{2h^3}{3} J_{02}B_{03}^1 + B_{03}^0 \int_{-h}^{h} x_3 j_2 \, dx_3 + B_{03}^1 \int_{-h}^{h} x_3^2 j_2 \, dx_3 - B_{02}^0 \int_{-h}^{h} x_3 j_2 \, dx_3 - B_{02}^1 \int_{-h}^{h} x_3^2 j_2 \, dx_3, \tag{61d}$$

$$\int_{-h}^{h} x_3 f_2 \, dx_3 = -\frac{2h^3}{3} J_{01}B_{03}^1 + B_{01}^0 \int_{-h}^{h} x_3 j_3 \, dx_3 + B_{01}^1 \int_{-h}^{h} x_3^2 j_3 \, dx_3 - B_{03}^0 \int_{-h}^{h} x_3 j_1 \, dx_3 - B_{03}^1 \int_{-h}^{h} x_3^2 j_1 \, dx_3. \tag{61e}$$

The expressions of the integrals in Eqs. (61) are provided in Appendix B.

5. A few remarks on the governing system

At this point, an account of the equations governing the magneto-thermo-elastokinetics of laminated composite, finitely electroconductive plates that incorporate the transverse shear deformations and the geometrical nonlinearities in von Kármán's sense should be given. For this purpose, one should remark that the whole theory is expressed in terms of 12 basic unknowns, namely, v_i, β_α, χ, ϑ, ψ, Θ_0, Θ_1 and γ_α ($i = \overline{1,3}$, $\alpha = 1,2$). The final governing system of the magneto-thermo-elastokinetic theory of laminated composite plates can be derived from Eqs. (19a–e) (elastic part), Eqs. (49) and (50) (electrodynamic part within the plate), Eqs. (59a, b) (thermal part) and Eqs. (38), (35a) (magnetic field outside the plate). This set of governing equations in conjunction with proper boundary and initial conditions should provide the solution of the problem formulated in this article.

Issues of the implementation of the numerical solution, implication of the transverse shear and the effects of the directionality properties/lay-ups of the constituent layers on the vibrational behavior will be investigated in the companion paper, Part 2.

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Appendix A: Definitions of Γ_1 and Γ_2 in Eqs. (48a, b) and of j_1, j_2, j_3

$$\begin{aligned}\Gamma_1 \equiv\ & \mathscr{M}_0[g_{11}](\dot{v}_2 B_{03}^0 - \dot{v}_3 B_{02}^0) + \mathscr{M}_0[g_{12}](\dot{v}_3 B_{01}^0 - \dot{v}_1 B_{03}^0)\\ &+ \mathscr{M}_1[g_{11}](\dot{v}_2 B_{03}^1 + \dot{\beta}_1 B_{03}^0 - \dot{v}_3 B_{02}^1) + \mathscr{M}_1[g_{12}](\dot{v}_3 B_{01}^1 - \dot{v}_1 B_{03}^1 + \dot{\beta}_2 B_{03}^0)\\ &+ \mathscr{M}_2[g_{11}]\dot{\beta}_1 B_{03}^1 + \mathscr{M}_2[g_{12}]\dot{\beta}_2 B_{03}^1,\end{aligned}$$

$$\begin{aligned}\Gamma_2 \equiv\ & -\mathscr{M}_0[g_{12}](\dot{v}_2 B_{03}^0 - \dot{v}_3 B_{02}^0) - \mathscr{M}_0[g_{22}](\dot{v}_3 B_{01}^0 - \dot{v}_1 B_{03}^0)\\ &- \mathscr{M}_1[g_{12}](\dot{v}_2 B_{03}^1 + \dot{\beta}_1 B_{03}^0 - \dot{v}_3 B_{02}^1) - \mathscr{M}_1[g_{22}](\dot{v}_3 B_{01}^1 - \dot{v}_1 B_{03}^1 + \dot{\beta}_2 B_{03}^0)\\ &- \mathscr{M}_2[g_{12}]\dot{\beta}_1 B_{03}^1 - \mathscr{M}_2[g_{22}]\dot{\beta}_2 B_{03}^1,\end{aligned}$$

$$\begin{aligned}j_1 =\ & \{g_{11}^{(i)}(\varphi + \dot{v}_2 B_{03}^0 - \dot{v}_3 B_{02}^0) + g_{12}^{(i)}(\psi + \dot{v}_3 B_{01}^0 - \dot{v}_1 B_{03}^0)\}\\ &+ x_3\{g_{11}^{(i)}(\dot{v}_2 B_{03}^1 + \dot{\beta}_2 B_{03}^0 - \dot{v}_3 B_{02}^1) + g_{12}^{(i)}(\dot{v}_3 B_{01}^1 - \dot{\beta}_1 B_{03}^0 - \dot{v}_1 B_{03}^1)\}\\ &+ x_3^2[g_{11}^{(i)}\dot{\beta}_2 - g_{12}^{(i)}\dot{\beta}_1]B_{03}^1,\end{aligned}$$

$$\begin{aligned}j_2 =\ & g_{12}^{(i)}(\varphi + \dot{v}_2 B_{03}^0 - \dot{v}_3 B_{02}^0) + g_{22}^{(i)}(\psi + \dot{v}_3 B_{01}^0 - \dot{v}_1 B_{03}^0)\\ &+ x_3\{g_{12}^{(i)}(\dot{v}_2 B_{03}^1 + \dot{\beta}_2 B_{03}^0 - \dot{v}_3 B_{02}^1) + g_{22}^{(i)}(\dot{v}_3 B_{01}^1 - \dot{\beta}_1 B_{03}^0 - \dot{v}_1 B_{03}^1)\}\\ &+ x_3^2[g_{12}^{(i)}\dot{\beta}_2 - g_{22}^{(i)}\dot{\beta}_1]B_{03}^1,\end{aligned}$$

$$\begin{aligned}j_3 =\ & h_{2,1} - h_{1,2} = h_{2,1}^{(i)} - h_{1,2}^{(i)} - \{g_{11}^{(i)}(\varphi_{,1} + \dot{v}_{2,1} B_{03}^0 - \dot{v}_{3,1} B_{02}^0)\\ &+ g_{12}^{(i)}(\psi_{,1} + \varphi_{,2} + \dot{v}_{3,1} B_{01}^0 + \dot{v}_{2,2} B_{03}^0 - \dot{v}_{1,1} B_{03}^0 - \dot{v}_{3,2} B_{02}^0)\\ &+ g_{22}^{(i)}(\psi_{,2} + \dot{v}_{3,2} B_{01}^0 - \dot{v}_{1,2} B_{03}^0)\}(x_3 - z_i)\\ &- \frac{x_3^2 - z_i^2}{2}\{g_{11}^{(i)}(\dot{v}_{2,1} B_{03}^1 + \dot{\beta}_{2,1} B_{03}^0 - \dot{v}_{3,1} B_{02}^1)\\ &+ g_{12}^{(i)}(\dot{v}_{3,1} B_{01}^1 - \dot{v}_{1,1} B_{03}^1 - \dot{\beta}_{1,1} B_{03}^0 + \dot{v}_{2,2} B_{03}^1 + \dot{\beta}_{2,2} B_{03}^0 - \dot{v}_{3,2} B_{02}^1)\\ &+ g_{22}^{(i)}(\dot{v}_{3,2} B_{01}^1 - \dot{v}_{1,2} B_{03}^1 - \dot{\beta}_{1,2} B_{03}^0)\}\end{aligned}$$

$$- \frac{x_3^3 - z_i^3}{3}\{g_{11}^{(i)}\dot{\beta}_{2,1} + g_{12}^{(i)}(\dot{\beta}_{1,1} + \dot{\beta}_{2,2}) + g_{22}^{(i)}\dot{\beta}_{1,2}\}B_{03}^1,$$

in which $x_3 \in [z_i, z_{i+1})$, $h_2^{(i)}$ $(i = \overline{1, N_L})$ are given by Eq. (52).

Appendix B: Definition of the integrals appearing in Eqs. (61a–e)

$$\int_{-h}^{h} h_1\,\mathrm{d}x_3 = h(h_1^+ + h_1^-) - \int_{-h}^{h} x_3 h_{1,3}\,\mathrm{d}x_3 = -\int_{-h}^{h} x_3 h_{1,3}\,\mathrm{d}x_3$$
$$= -2h\{\mathscr{M}_2[g_{12}](\dot{v}_2 B_{03}^1 + \dot{\beta}_2 B_{03}^0 - \dot{v}_3 B_{02}^1) + \mathscr{M}_2[g_{22}](\dot{v}_3 B_{01}^1 - \dot{v}_1 B_{03}^1 - \dot{\beta}_1 B_{03}^0)\},$$

in which Eq. (35a) is invoked. Similarly,

$$\int_{-h}^{h} h_2\,\mathrm{d}x_3 = 2h\{\mathscr{M}_2[g_{11}](\dot{v}_2 B_{03}^1 + \dot{\beta}_2 B_{03}^0 - \dot{v}_3 B_{02}^1) + \mathscr{M}_2[g_{12}](\dot{v}_3 B_{01}^1 - \dot{v}_1 B_{03}^1 - \dot{\beta}_1 B_{03}^0)\},$$

$$\int_{-h}^{h} j_1\,\mathrm{d}x_3 = 2h\{[\mathscr{M}_0[g_{11}](\varphi + \dot{v}_2 B_{03}^0 - \dot{v}_3 B_{02}^0) + \mathscr{M}_0[g_{12}](\psi + \dot{v}_3 B_{01}^0 - \dot{v}_1 B_{03}^0)]$$
$$+ \mathscr{M}_2[g_{11}]\dot{\beta}_2 B_{03}^1 - \mathscr{M}_0[g_{12}]\dot{\beta}_1 B_{03}^1\},$$

$$\int_{-h}^{h} x_3 j_1\,\mathrm{d}x_3 = 2h\{\mathscr{M}_2[g_{11}](\dot{v}_2 B_{03}^1 + \dot{\beta}_2 B_{03}^0 - \dot{v}_3 B_{02}^1) + \mathscr{M}_2[g_{12}](\dot{v}_3 B_{01}^1 - \dot{\beta}_1 B_{03}^0 - \dot{v}_1 B_{03}^1)\},$$

$$\int_{-h}^{h} j_2\,\mathrm{d}x_3 = 2h\{[\mathscr{M}_0[g_{12}](\varphi + \dot{v}_2 B_{03}^0 - \dot{v}_3 B_{02}^0) + \mathscr{M}_0[g_{22}](\psi + \dot{v}_3 B_{01}^0 - \dot{v}_1 B_{03}^0)]$$
$$+ \mathscr{M}_2[g_{12}]\dot{\beta}_2 B_{03}^1 - \mathscr{M}_2[g_{22}]\dot{\beta}_1 B_{03}^1\},$$

$$\int_{-h}^{h} x_3 j_2\,\mathrm{d}x_3 = 2h\{\mathscr{M}_2[g_{12}](\dot{v}_2 B_{03}^1 + \dot{\beta}_2 B_{03}^0 - \dot{v}_3 B_{02}^1) + \mathscr{M}_2[g_{22}](\dot{v}_3 B_{01}^1 - \dot{\beta}_1 B_{03}^0 - \dot{v}_1 B_{03}^1)\},$$

$$\int_{-h}^{h} j_3\,\mathrm{d}x_3 = h(j_{s3}^+ + j_{s3}^-) - \int_{-h}^{h} x_3 j_{3,3}\,\mathrm{d}x_3 = \int_{-h}^{h} x_3[h_{1,32} - h_{2,31}]\,\mathrm{d}x_3$$
$$= 2h\{\mathscr{M}_2[g_{12}](\ddot{v}_2 B_{03}^1 + \ddot{\beta}_2 B_{03}^0 - \ddot{v}_3 B_{02}^1) + \mathscr{M}_2[g_{22}](\ddot{v}_3 B_{01}^1 - \ddot{\beta}_1 B_{03}^0 - \ddot{v}_1 B_{03}^1)$$
$$+ \mathscr{M}_2[g_{11}](\ddot{v}_2 B_{03}^1 + \ddot{\beta}_2 B_{03}^0 - \ddot{v}_3 B_{02}^1) + \mathscr{M}_2[g_{12}](\ddot{v}_3 B_{01}^1 - \ddot{\beta}_1 B_{03}^0 - \ddot{v}_1 B_{03}^1)\}.$$

It is recalled that in deriving the above equation, the induced surface current $\mathbf{J}_s = 0$. As a result, $j_{s3}^{+} = j_{s3}^{-} = 0$.

$$\begin{aligned}\int_{-h}^{h} x_3 j_3 \,\mathrm{d}x_3 &= \frac{1}{2}\int_{-h}^{h} x_3^2[h_{1,32} - h_{2,31}]\,\mathrm{d}x_3 \\ &= h\{\mathscr{M}_2[g_{12}][\varphi_{,2} + (\dot{v}_{2,2}B_{03}^0 - \dot{v}_{3,2}B_{02}^0)] + \mathscr{M}_2[g_{22}][\psi_{,2} + (\dot{v}_{3,2}B_{01}^0 - \dot{v}_{1,2}B_{03}^0)] \\ &\quad + \mathscr{M}_2[g_{11}][\varphi_{,1} + (\dot{v}_{2,1}B_{03}^0 - \dot{v}_{3,1}B_{02}^0)] + \mathscr{M}_2[g_{12}][\psi_{,1} + (\dot{v}_{3,1}B_{01}^0 - \dot{v}_{1,1}B_{03}^0)] \\ &\quad + \mathscr{M}_4[g_{12}]\dot{\beta}_{2,2}B_{03}^1 - \mathscr{M}_4[g_{22}]\dot{\beta}_{1,2}B_{03}^1 + \mathscr{M}_4[g_{11}]\dot{\beta}_{2,1}B_{03}^1 - \mathscr{M}_4[g_{12}]\dot{\beta}_{1,1}B_{03}^1\},\end{aligned}$$

$$\begin{aligned}\int_{-h}^{h} x_3 h_1 \,\mathrm{d}x_3 = &\frac{1}{2}h^2\gamma_2 - \frac{h^3}{3}\chi_{,1} - h\{\mathscr{M}_2[g_{12}](\varphi + \dot{v}_2B_{03}^0 - \dot{v}_3B_{02}^0) \\ &+ \mathscr{M}_2[g_{22}](\psi + \dot{v}_3B_{01}^0 - \dot{v}_1B_{03}^0)\} - h\{\mathscr{M}_4[g_{12}]\dot{\beta}_2B_{03}^1 - \mathscr{M}_4[g_{22}]\dot{\beta}_1B_{03}^1\},\end{aligned}$$

$$\begin{aligned}\int_{-h}^{h} x_3 h_2 \,\mathrm{d}x_3 = &-\frac{1}{2}h^2\gamma_1 - \frac{h^3}{3}\chi_{,2} + h\{\mathscr{M}_4[g_{11}](\varphi + \dot{v}_2B_{03}^0 - \dot{v}_3B_{02}^0) \\ &+ \mathscr{M}_4[g_{12}](\psi + \dot{v}_3B_{01}^0 - \dot{v}_1B_{03}^0)\} + h\{\mathscr{M}_4[g_{11}]\dot{\beta}_2B_{03}^1 - \mathscr{M}_4[g_{12}]\dot{\beta}_1B_{03}^1\},\end{aligned}$$

$$\begin{aligned}\int_{-h}^{h} x_3^2 j_1 \,\mathrm{d}x_3 = &\,2h\{\mathscr{M}_2[g_{11}](\varphi + \dot{v}_2B_{03}^0 - \dot{v}_3B_{02}^0) + \mathscr{M}_2[g_{12}](\psi + \dot{v}_3B_{01}^0 - \dot{v}_1B_{03}^0)\} \\ &+ 2h\{\mathscr{M}_4[g_{11}]\dot{\beta}_2B_{03}^1 - \mathscr{M}_4[g_{12}]\dot{\beta}_1B_{03}^1\},\end{aligned}$$

$$\begin{aligned}\int_{-h}^{h} x_3^2 j_2 \,\mathrm{d}x_3 = &\,2h\{\mathscr{M}_2[g_{12}](\varphi + \dot{v}_2B_{03}^0 - \dot{v}_3B_{02}^0) + \mathscr{M}_2[g_{22}](\psi + \dot{v}_3B_{01}^0 - \dot{v}_1B_{03}^0)\} \\ &+ 2h\{\mathscr{M}_4[g_{12}]\dot{\beta}_2B_{03}^1 - \mathscr{M}_4[g_{22}]\dot{\beta}_1B_{03}^1\}.\end{aligned}$$

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